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Coherent Images of Conventional Targets

This memo describes the images of several conventional targets formed with diffraction limited optics and coherent illumination. The targets are variable density targets in one dimension.

They include:

A. Periodic targets
   1. sine waves
   2. square wave gratings

B. Non-periodic targets
   1. edge
   2. line
   3. 3 bars

The image light intensity for coherent illumination is given by:[1]

$$I(x) = \left|\int T_c(\omega)A(\omega)e^{j\omega x}\, d\omega\right|^2 \qquad (1)$$

$T_c(\omega)$ ($f(\omega)$ in ref. 1) is the coherent transfer function. $A(\omega)$ is the Fourier transform of the target electric field transmission, $a(x)$.

For diffraction limited optics:

$$T_c(\omega) = P_\Omega(\omega) = \begin{cases} 1 & |\omega| < \Omega \\ 0 & |\omega| > \Omega \end{cases} \qquad (2)$$

$$a(x) \longleftrightarrow A(\omega) \qquad (3)$$

where Ω is the coherent cutoff radian spatial frequency.

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The test targets are divided into two groups, periodic and non-periodic targets. A periodic target should have about eight or more cycles so that the spectrum is almost discrete. The spectrum is confined primarily to the region of the harmonics and may be approximated by a series of delta functions. A non-periodic target has a continuous spectrum.

As Ω is increased the nature of the image of a periodic target changes discretely everytime another harmonic is passed; the image of a non-periodic target changes continuously.

A. Periodic targets

1. Sine waves

Variable density sine wave targets may be sinusoidal in electric field transmission with:

$$a_1(x) = \frac{1}{2}\left[1 + m \cos x \right] \tag{4}$$

Target modulation is m.

$$A_1(\omega) = \frac{1}{2}\left[\delta(\omega) + \frac{m}{2}\ \delta(\omega+1) + \frac{m}{2}\ \delta(\omega-1) \right] \tag{5}$$

However, sine wave targets normally used with incoherent light are sinusoidal in light transmission.

$$t_2(x) = \frac{1}{2}\ (1 + m \cos x) \tag{6}$$

For m = 1 the electric field transmission is:

$$a_2(x) = \left[t_2(x) \right]^{1/2} = \left| \cos \frac{x}{2} \right| = \tag{7}$$

$$= \frac{2}{\pi}\left[1 + \frac{2}{3} \cos x - \frac{2}{15} \cos 2x + \frac{2}{35} \cos 3x \ldots \right].$$

$$A_2(\omega) = \frac{2}{\pi} \sum_{n=-\infty}^{\infty} (-1)^n \left(\frac{1}{1-4n^2} \right) \delta(\omega-n) \tag{8}$$

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$I(x)$ may be calculated from Eq. (1).

The reduction in intensity modulation is:

$$M = \frac{1}{m} \left[\frac{I(\text{maximum}) - I(\text{minimum})}{I(\text{maximum}) + I(\text{minimum})} \right] \tag{9}$$

The letter M is used in Eq. 9 instead of $T(k)$ because Eq. 9 is not the coherent transfer function. Figure 1 shows M for sine wave targets in electric field and light intensity transmission (Eq. 4 and 6) for $m = 1$ and $m \ll 1$. For Eq. 6 M is a function of m. This effect will be discussed in a future memo. Note that $K_o = 1/2\pi\ \Omega$ and that Figure 1 is drawn for $K_o = 12\ \ell/\text{mm}$.
The transmission of the following targets are binary in nature so that $a(x)$ is proportional to $t(x)$. The expressions are given for 100 per-cent modulation ($m = 1$).

2. Square wave targets

$$a_3(x) = \frac{1}{2} - \frac{2}{\pi} \cos x + \frac{2}{3\pi} \cos 3x - \frac{2}{5\pi} \cos 5x \ldots \tag{10}$$

$$A_3(\omega) = \frac{1}{2} - \frac{1}{\pi} \sum_{n=-\infty}^{\infty} (-1)^n \frac{1}{2n+1}\ \delta\ (\omega - 2n + 1) \tag{11}$$

Figure 2 illustrates the image of a square wave grating for $1 < \Omega < 7$. In Eq. 10 the fundamental frequency is greater than the bias term. This produces the multiple frequency "ringing" or fringe pattern shown in Figure 2.

B. <u>Non-Periodic Targets</u>

1. Edge

$$a_4(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases} \tag{12}$$

$$I_4(x) = \left[1 + \frac{2}{\pi} S_i(\Omega x) \right]^2 \tag{13}$$

$$S_i(\Omega x) = \int_0^{\Omega x} \frac{\sin v}{v}\, dv \text{ is the sine integral.}^2 \tag{14}$$

Eq. 13 is shown in Figure 3. While the peak amplitude of the "ringing" does not vanish even for large values of Ω the width of the "ringing" does. This "ringing" is called Gibbs' phenomenon.[3] Gibbs' phenomenon arises from the sharp cutoff frequency of the optical system. The amplitude of the "ringing" may be decreased only by apodization.

2. Line

$$a_5(x) = \delta(x) \tag{15}$$

$$I_5(x) = \left[\frac{\sin \Omega x}{\Omega x}\right]^2 \tag{16}$$

The image of a line is shown in Figure 4.

3. Three Bars

The coherent image of a three bar target is particularly interesting. The transmission of a three bar target on a light background may be written as a sum of rectangular pulses. The width of one bar is 2d.

$$a_6(x) = 1 - [P_{5d}(x) - P_{3d}(x) + P_d(x)] \tag{17}$$

$$A_6(\omega) = \delta(\omega) - (1 + 2 \cos 4\omega d) \frac{\sin \omega d}{\omega d} \tag{18}$$

The continuous spectrum of a three bar target (Eq. 18 with $d = \pi/2$) and the discrete spectrum of a periodic square wave grating (Eq. 11) are shown in Figure 5. The image of a three bar target on a light background is:

$$I_6(x) = [1 - E(x)]^2 \tag{19}$$

$$\begin{aligned} E(x) = 1/\pi \{ & [S_i(\Omega(x+5d)) - S_i(\Omega(x-5d))] \\ & -[S_i(\Omega(x+3d)) - S_i(\Omega(x-3d))] \\ & +[S_i(\Omega(x+d)) - S_i(\Omega(x-d))]\} \end{aligned} \tag{20}$$

As Ω decreases the image of the three bars vanishes gradually because $A_6(\omega)$ is continuous, not discrete. Figure 6 shows the modulation for a coherent cutoff frequency of 10 cycles per millimeter. A three bar target will still have 37 % modulation at 11.4 cycles per millimeter. The frequency at which the modulation drops to 4% (the limit of visibility) was not calculated, but lies between 11.4 and 13.3 cycles per millimeter. Thus with coherent light a three bar target can be resolved as three bars at a frequency 15 to 30% above the coherent cutoff frequency. [redacted] has calculated the images of two and three bar targets for incoherent light. For incoherent light in each case the modulation of the bars vanishes sharply within a few percent of the incoherent cutoff frequency. With coherent light when the three bars vanish, the target appears as two clearly visible cycles from 13.3 to 20 cycles per millimeter.

Figure 7 shows the right half of the image of a 10 cycle/millimeter three bar target as a function of the optical cutoff frequency K_o. Three bars may be seen for $K_o < 10$ cycles/millimeter in Figure 7c. The peak intensities move closer together for larger values of K_o. The image is almost unchanged for $11 < K_o < 28$ cycles per millimeter. At three times the cutoff frequency ($K_o = 30$ cycles per millimeter), the peak starts to show "ringing".

These results are presented to give the diffraction limited images of conventional incoherent targets. These targets are not optimum tests for a coherent imagery. Better test targets, variable area test targets and the effect of aberration will be considered later.

Note

In the figures Ω, the radian cutoff frequency has been replaced by $K_o = 1/2\pi\ \Omega$. Figures 1 to 5 inclusive are not shown to scale; small amplitudes have been magnified.

[redacted]

JLK/mq

Acknowledgement: Figures 6 and 7 were computed from Eqs. 19 and 20 [redacted]
[redacted]

References

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1. [redacted]

2. [redacted]

3. Papoulis, A., "The Fourier Integral and Its Applications", p. 30-33, McGraw-Hill, New York, 1962

4. Sokolnikoff, I.S.; Redheffer, R.M., "Mathematics of Physics and Modern Engineering", p. 181 Fig. 1G, McGraw-Hill Book Co, Inc. New York, 1958.

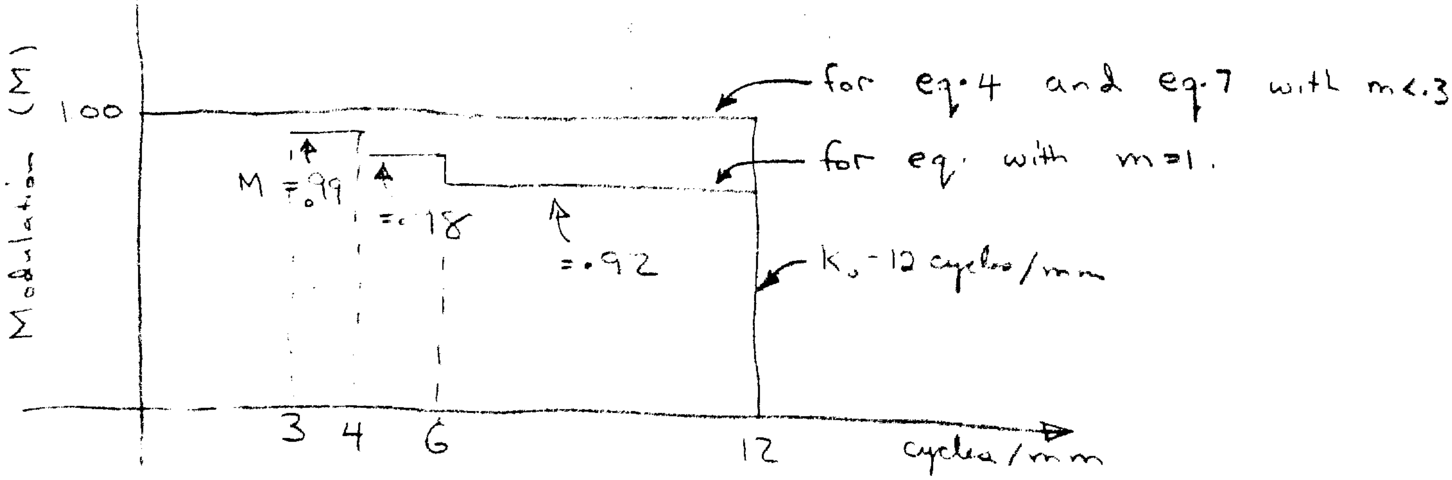

Figure 1. Image Modulation Reduction for sine wave targets.

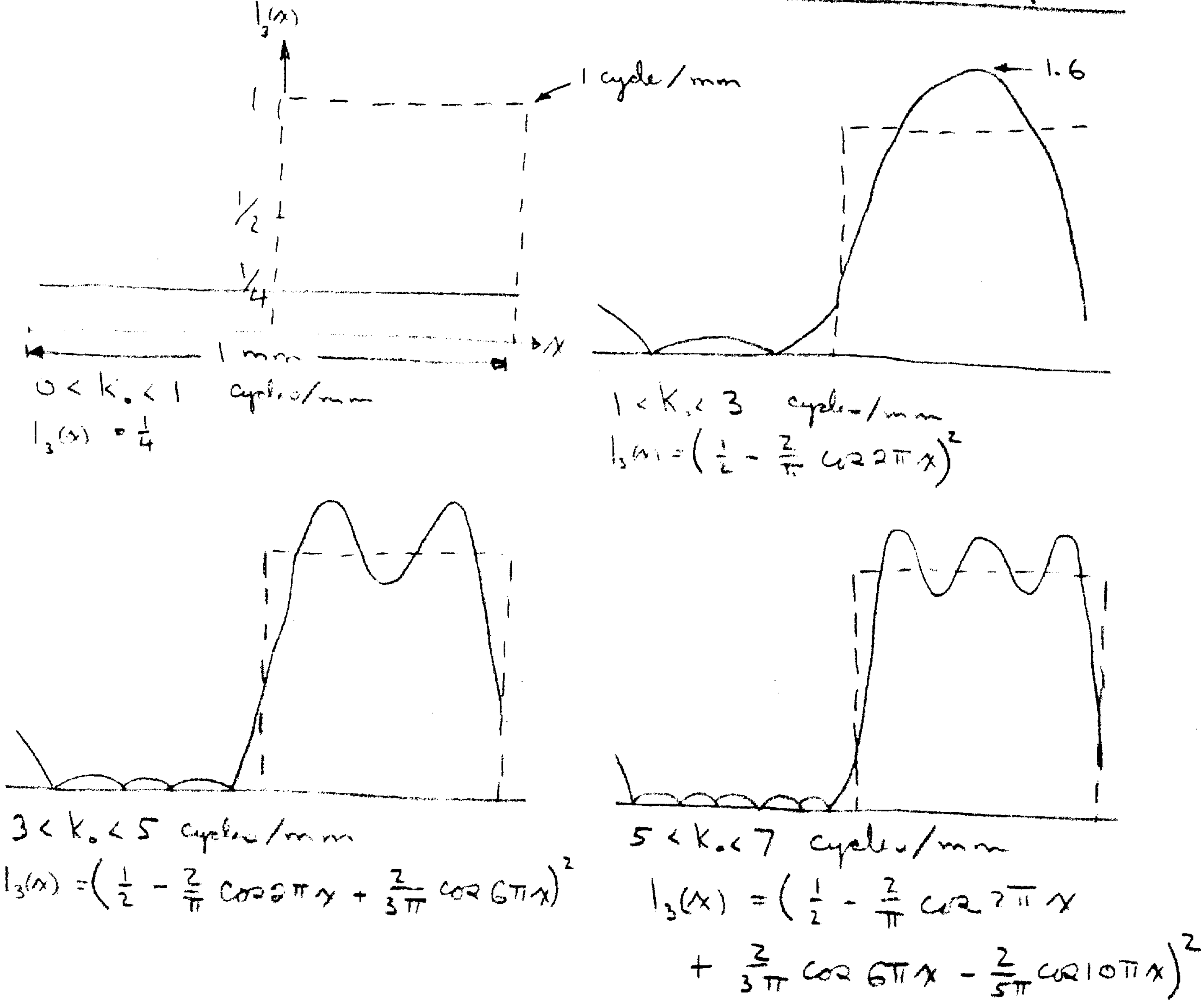

Figure 2. Square Wave Grating Images [4]

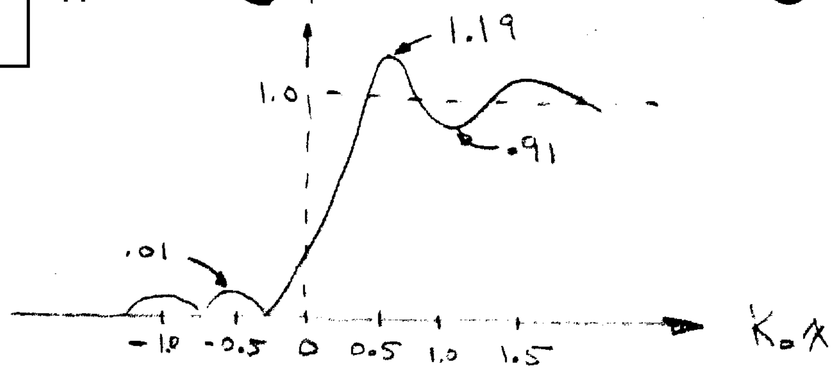

Figure 3 Edge Image

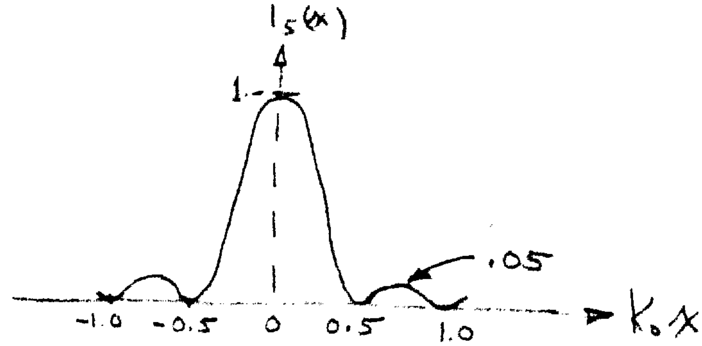

Figure 4 Bright Line Image

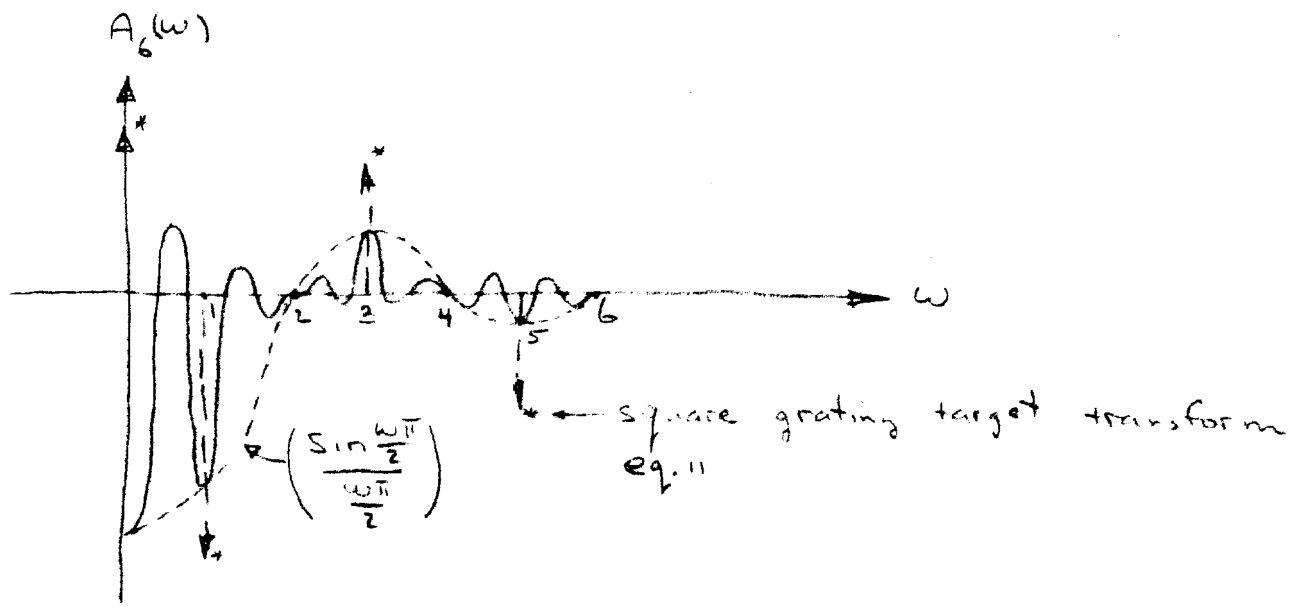

Figure 5 Fourier Transform of a Three Bar Target on a Bright Background

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